APPLIED SCIENCE

The International Journal of Applied Sciences.
Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
https://international-journal-of-applied-sciences.jo

Deep Learning-Assisted Robust Joint Estimation of Array Amplitude–Phase Errors and Direction of Arrival Using Auxiliary Elements

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DOI: https://doi.org/10.64440/IJAS/IJAS10011

ARTICLE INFO

Article history
Received Mar 14, 2026
Revised Mar 15, 2026
Accepted May 16, 2026

Keywords
sensor gain–phase error;
direction-of-arrival estimation;
DOA estimation;
auxiliary array elements;
array calibration;
deep learning;
subspace methods;
robust estimation;
array signal processing.

ABSTRACT

Array imperfections are unavoidable in practical sensing systems and can significantly deteriorate the accuracy of direction-of-arrival (DOA) estimation. Among the different forms of array perturbation, sensor gain and phase errors are particularly common because of manufacturing tolerances, receiver-chain mismatch, aging, temperature fluctuations, and imperfect calibration. To address the degradation of DOA estimation caused by gain–phase errors, this study develops a deep learning-assisted robust joint estimation framework for simultaneously estimating sensor gain–phase errors and the DOA of incident signals. A small number of accurately calibrated auxiliary array elements are introduced at one side of the array to impose a structured constraint on the gain–phase error matrix. Based on this structure, a transformation matrix is constructed and integrated with a subspace-based estimation framework to jointly recover the sensor errors and source directions. To further improve robustness under unequal source powers and low signal-to-noise ratio (SNR) conditions, a deep learning module is incorporated to learn the nonlinear relationship between distorted array observations, calibration parameters, and DOA information. The proposed framework therefore combines the interpretability of model-based subspace estimation with the adaptability of data-driven learning. In particular, the auxiliary elements improve parameter identifiability, while the learning-assisted refinement reduces the sensitivity of the estimator to inaccurate covariance estimation and large power disparities among incident sources. The proposed method is evaluated through numerical simulations under different SNR levels, source-power ratios, numbers of snapshots, gain–phase error levels, and numbers of auxiliary elements. The results demonstrate that the proposed strategy can achieve accurate simultaneous estimation of array errors and DOAs while maintaining improved robustness in nonideal sensing environments. The method provides a practical approach for array calibration and intelligent spatial sensing in radar, wireless communication, localization, and other applied sensing systems.

1.INTRODUCTION

Array perturbations are common nonidealities in practical sensor-array systems and can substantially degrade the performance of spatial signal-processing algorithms [1–3]. In real-world applications, deviations from the ideal array model may arise during sensor fabrication, assembly, installation, operation, and long-term use. According to their physical origins, array perturbations can generally be classified into three major categories: **mutual coupling between array elements** [4,5], **sensor gain and phase errors** [6,7], and **sensor-position errors** [8,9]. Among these perturbations, gain–phase errors are particularly prevalent because individual receiving channels rarely exhibit perfectly identical amplitude responses and phase characteristics. Consequently, gain–phase calibration has become an important research problem in high-resolution direction-of-arrival (DOA) estimation.

Accurate DOA estimation is essential for numerous applied sensing systems, including radar target localization, wireless communication, passive source localization, acoustic sensing, navigation, and intelligent sensor networks. Most high-resolution DOA estimation methods, such as MUSIC and ESPRIT, rely on an accurate array manifold. When the actual gain and phase responses of individual sensors deviate from their nominal values, the steering vectors used by these algorithms become mismatched with the actual received data. This model mismatch can lead to biased angular estimates, spurious spectral peaks, source ambiguity, and, under severe conditions, failure of source resolution.

Early approaches to gain–phase calibration generally required a calibrated reference source as prior information [10,11]. Although these methods can provide reliable calibration under controlled conditions, the requirement for a known calibration source limits their flexibility in practical environments. To eliminate this dependence, self-calibration techniques based on subspace methods were subsequently developed. For example, several studies [12,13] proposed iterative procedures in which the DOAs and sensor gain–phase errors are alternately estimated. Through repeated iterations, the estimated parameters gradually converge toward their true values. Although such approaches realize self-calibration without requiring a dedicated calibration source, their repeated optimization and matrix decomposition operations lead to relatively high computational complexity, particularly when the number of sensors or sources increases.

To avoid iterative estimation, Paulraj and Kailath exploited the Toeplitz structure of the covariance matrix of the received signals in a uniform linear array (ULA) and developed a subspace-transformation-based method for estimating gain–phase errors [14]. This approach provides an important theoretical foundation for noniterative array calibration. However, the method requires calibration of multiple diagonals of the upper triangular portion of the covariance matrix, resulting in a relatively complicated implementation and considerable computational burden. To overcome this limitation, a simplified calibration approach based on different diagonals of the covariance matrix was subsequently proposed [15], substantially reducing the computational requirements. Nevertheless, these methods are primarily applicable to uniform linear arrays and therefore remain subject to array-geometry constraints.

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Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
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To address the limitations associated with specific array configurations, a subspace-based gain–phase error calibration method for two-dimensional arrays was reported in [7]. Despite these advances, existing approaches generally face one or more of the following limitations: relatively high computational complexity, sensitivity to noise and finite-snapshot effects, dependence on particular array geometries, and inability to estimate gain–phase errors and DOAs accurately and simultaneously. These limitations become more pronounced when multiple sources have substantially different powers. In such situations, weak sources can contribute less reliably to the estimated covariance matrix and noise subspace, which can subsequently degrade the calibration accuracy.

Recent developments in machine learning provide an additional opportunity to improve the robustness of array signal processing. Unlike purely model-based methods, deep neural networks can learn nonlinear relationships between distorted array observations and latent parameters from representative training data. When appropriately integrated with physical array models, deep learning can therefore provide a complementary mechanism for compensating for model mismatch and improving estimation robustness. However, a purely data-driven estimator may lack physical interpretability and may exhibit reduced generalization when the operating conditions differ substantially from the training distribution. Therefore, a **hybrid model-driven and data-driven architecture** is particularly attractive for practical array calibration.

In this work, a small number of accurately calibrated **auxiliary array elements** are introduced at one end of the array. These elements impose repeated and known entries in the gain–phase error matrix, allowing its structure to be transformed into a form that facilitates joint estimation. Based on the resulting structured error matrix, a transformation matrix is constructed and combined with a subspace-based spatial estimation procedure. A deep learning-assisted refinement stage is further introduced to improve estimation under unfavorable conditions, particularly when the incident sources have substantially different powers or when the covariance matrix is estimated from a limited number of snapshots.

The main contributions of this study are summarized as follows:

1. **A structured auxiliary-element calibration mechanism** is established by introducing a small number of accurately calibrated sensors at one side of the array. This reduces the number of unknown gain–phase parameters while retaining the spatial information required for DOA estimation.
2. **A joint model-based estimation framework** is developed to simultaneously estimate the sensor gain–phase errors and the DOAs of multiple incident sources without requiring a conventional external calibration source.
3. **A deep learning-assisted refinement mechanism** is incorporated into the estimation framework. The learning model is designed to exploit the statistical characteristics of the distorted array observations and refine the initial subspace-based estimates.

4. **Robustness to unequal source powers** is explicitly considered. This is important because large power differences between incident sources can make some source-dependent calibration estimates substantially less reliable than others.
5. **A comprehensive simulation framework** is established to evaluate the proposed method under different SNRs, source-power ratios, snapshot numbers, gain–phase error levels, source angular separations, and numbers of auxiliary elements.

The proposed approach therefore combines the physical interpretability and mathematical structure of subspace methods with the nonlinear approximation capability of deep learning, providing a practical framework for robust intelligent array calibration and DOA estimation.

2. Signal Model

2.1. Uniform Linear Array Signal Model

Consider an ideal uniform linear array (ULA) consisting of (M) sensors with an inter-element spacing of half a wavelength. Thus,

$$d = \frac{\lambda}{2},$$

where (d) denotes the inter-element spacing and λ is the wavelength of the received signal.

Assume that (L) independent far-field narrowband signals impinge on the ULA from directions $\theta_1,\theta_2,\ldots,\theta_L$ as illustrated in **Figure 1**. The steering vector corresponding to the (l)-th incident signal can be expressed as

$$\mathbf{a}(\theta_l) = \left[1, e^{-j\varphi_l}, e^{-j2\varphi_l}, \ldots, e^{-j(M-1)\varphi_l}\right]^T, \tag{1}$$

where

$$\varphi_l = \frac{2\pi d \sin(\theta_l)}{\lambda}, \qquad l = 1, 2, \ldots, L.$$

Here, θ_l denotes the DOA of the l-th source.

Let (s_l(k)) represent the waveform of the l-th source at snapshot (k). Assuming that the incident signals are mutually independent, the received array observation under ideal sensor conditions can be written as

$$\mathbf{x}(k) = \sum_{l=1}^{L} \mathbf{a}(\theta_l) s_l(k) + \mathbf{n}(k), \tag{2}$$

where

$$\mathbf{x}(k) \in \mathbb{C}^{M \times 1}$$

is the received data vector and

$$\mathbf{n}(k) \in \mathbb{C}^{M \times 1}$$

denotes the additive noise vector. The noise is assumed to be zero-mean additive white Gaussian noise (AWGN) and statistically independent of the incident signals.

The theoretical covariance matrix of the received signal is given by

$$\mathbf{R} = E\left[\mathbf{x}(k)\mathbf{x}^H(k)\right],$$

which can be expanded as

$$\mathbf{R} = \sum_{l=1}^{L} \mathbf{a}(\theta_l) E\left[s_l(k) s_l^*(k)\right] \mathbf{a}^H(\theta_l) + \sigma_n^2 \mathbf{I}.$$

Because the incident signals are mutually independent,

$$E[s_l(k) s_l^*(k)] = \sigma_l^2,$$

where σ_l^2 is the power of the (l)-th source. Therefore,

$$\mathbf{R} = \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l) + \sigma_n^2 \mathbf{I}, \tag{3}$$

where σ_n^2 denotes the noise power and I is an (M × M) identity matrix.

In practical experiments, the theoretical covariance matrix is unavailable and must be estimated from a finite number of snapshots. Accordingly, the theoretical covariance matrix is replaced by the **sample covariance matrix (SCM)**,

$$\widehat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k), \tag{4}$$

where (K) is the total number of available snapshots.

The accuracy of $\widehat{\mathbf{R}}$ depends strongly on the number of snapshots and the SNR. This finite-sample effect becomes particularly important when the proposed algorithm is applied to weak sources or when substantial gain–phase errors are present.

2.2. Signal Model with Sensor Gain–Phase Errors

In an ideal ULA, all sensors are assumed to have identical amplitude and phase responses. In practical systems, however, the receiving channels exhibit individual gain and phase deviations. Let the amplitude and phase errors associated with the (m)-th sensor be denoted by α_m and ϕ_m, respectively.

The amplitude-error matrix can then be represented by

$$\boldsymbol{\Gamma} = \operatorname{diag}\left\{\alpha_1, \alpha_2, \ldots, \alpha_M\right\}, \tag{5}$$

while the phase-error matrix is expressed as

$$\boldsymbol{\Phi} = \operatorname{diag}\left\{e^{j\pi\phi_1}, e^{j\pi\phi_2}, \ldots, e^{j\pi\phi_M}\right\}. \tag{6}$$

Here, {diag}{.}) denotes the diagonalization operator.

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The International Journal of Applied Sciences.
Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
https://international-journal-of-applied-sciences.jo

To eliminate the inherent scale ambiguity between source amplitudes and sensor gains, the first sensor is conventionally selected as the reference sensor. Thus,

$$\alpha_1 = 1, \qquad \phi_1 = 0.$$

The combined gain–phase error matrix is defined as

$$\boldsymbol{\Psi} = \boldsymbol{\Phi}\boldsymbol{\Gamma}.$$

Consequently, the received signal in the presence of sensor gain–phase errors becomes

$$\tilde{\mathbf{x}}(k) = \boldsymbol{\Phi}\boldsymbol{\Gamma}\sum_{l=1}^{L}\mathbf{a}(\theta_l)s_l(k) + \mathbf{n}(k). \tag{7}$$

The corresponding distorted steering vector is therefore

$$\tilde{\mathbf{a}}(\theta) = \boldsymbol{\Phi}\boldsymbol{\Gamma}\mathbf{a}(\theta) = \boldsymbol{\Psi}\mathbf{a}(\theta). \tag{8}$$

Equation (8) illustrates the fundamental difficulty addressed in this study. The actual steering vector is no longer determined exclusively by the source direction. Instead, it is jointly affected by the unknown sensor-dependent amplitude and phase responses. Consequently, conventional DOA algorithms based on the ideal steering vector may suffer from significant estimation errors.

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3. Proposed Joint Estimation Framework

3.1. Structured Error Matrix Induced by Auxiliary Elements

According to matrix decomposition, Equation (8) can be rewritten as

$$\boldsymbol{\Psi}\mathbf{a}(\theta) = \mathbf{T}_{\varphi}(\theta)\boldsymbol{\varphi}, \tag{9}$$

Where

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$$\boldsymbol{\Psi} = \boldsymbol{\Phi}\boldsymbol{\Gamma}$$

denotes the combined gain–phase error matrix and

$$\boldsymbol{\varphi} = \mathrm{diag}(\boldsymbol{\Psi})$$

represents the gain–phase error vector.

The matrix

$$\mathbf{T}_{\varphi}(\theta) \in \mathbb{C}^{M\times M}$$

is constructed according to the repeated elements of the gain–phase error matrix. Specifically, its (m)-th column can be expressed as

$$\mathbf{E}_m\mathbf{a}(\theta),$$

where E_m is a selection matrix whose elements satisfy

$$[\mathbf{E}_m]_{pq} = \begin{cases} 1, & [\boldsymbol{\Psi}]_{pq} = [\boldsymbol{\varphi}]_m, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

The subspace principle provides an important property for estimating the unknown error parameters. Let $\mathbf{U}_n$ denote the noise-subspace eigenvectors obtained from the eigendecomposition of the received covariance matrix. In the presence of gain–phase errors, the following relationship can be established:

$$\boldsymbol{\varphi}^H\mathbf{Q}_{\varphi}(\theta)\boldsymbol{\varphi} = 0, \tag{11}$$

where

$$\mathbf{Q}_{\varphi}(\theta) = \mathbf{T}_{\varphi}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{T}_{\varphi}(\theta).$$

Since

$$\mathrm{rank}(\mathbf{U}_n\mathbf{U}_n^H) = M - L,$$

the matrix $\mathbf{Q}_{\varphi}(\theta)$ exhibits singular behavior at the actual source directions. This property provides the theoretical basis for jointly estimating the gain–phase errors and DOAs.

Auxiliary calibrated elements

A key feature of the proposed method is the introduction of a small number of accurately calibrated auxiliary elements at one end of the array. Rather than requiring every sensor to be calibrated independently, only a limited number of elements are assumed to have accurately known amplitude and phase responses.

Let (P-1) additional calibrated elements be introduced. The first (P) elements are therefore treated as calibrated references, giving

$$\alpha_m = 1, \qquad m = 1, 2, \ldots, P, \tag{12}$$

And

$$\phi_m = 0, \qquad m = 1, 2, \ldots, P. \tag{13}$$

The gain and phase matrices can consequently be partitioned as

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{I}_{P-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_P \end{bmatrix}, \tag{14}$$

and

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{P-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_P \end{bmatrix}, \tag{15}$$

where

$$\mathbf{\Gamma}_P = \mathrm{diag}\{\alpha_P, \alpha_{P+1}, \ldots, \alpha_M\},$$

and

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$$\mathbf{\Phi}_P = \mathrm{diag}\{e^{j\pi\phi_P}, e^{j\pi\phi_{P+1}}, \dots, e^{j\pi\phi_M}\}.$$

The combined error matrix can then be written in block form as

$$\mathbf{\Psi} = \mathbf{\Gamma}\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{P-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Psi}_P \end{bmatrix}. \tag{16}$$

Accordingly, the unknown gain–phase error vector becomes

$$\begin{aligned} \boldsymbol{\varphi}_P &= \mathrm{diag}(\mathbf{\Psi}_P) \\ &= \left[\alpha_P e^{j\pi\phi_P}, \alpha_{P+1} e^{j\pi\phi_{P+1}}, \dots, \alpha_M e^{j\pi\phi_M}\right]^T. \end{aligned} \tag{17}$$

This structural transformation is important because it reduces the number of independent error parameters and increases the identifiability of the unknown sensor responses.

3.2. Reduced-Dimensional Transformation Matrix

Based on the structured error matrix, Equation (10) can be reformulated using only the distinct unknown gain–phase coefficients:

$$[\mathbf{E}_{\varphi_g}]_{mn} = \begin{cases} 1, & [\mathbf{\Psi}]_{mn} = (\boldsymbol{\varphi}_P)_g, \\ 0, & \text{otherwise}, \end{cases} \qquad g = 1, 2, \dots, G, \tag{18}$$

where

$$G = M - P + 1.$$

The corresponding transformation matrix becomes

$$\mathbf{T}_{\varphi}(\theta) = [\mathbf{E}_{\varphi_1}\mathbf{a}(\theta), \mathbf{E}_{\varphi_2}\mathbf{a}(\theta), \dots, \mathbf{E}_{\varphi_G}\mathbf{a}(\theta)] \in \mathbb{C}^{M\times G}. \tag{19}$$

Equation (11) can consequently be reduced to

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$$\varphi_P^H \mathbf{Q}_\varphi(\theta)\varphi_P = 0, \tag{20}$$

Where

$$\mathbf{Q}_\varphi(\theta) \in \mathbb{C}^{G\times G}.$$

When

$$G \leq M - L,$$

which is equivalent to

$$L \leq P - 1,$$

the matrix $\mathbf{Q}_\varphi(\theta)$ is generally full rank for arbitrary angles that do not correspond to actual source directions. At an actual DOA,

$$\theta = \theta_l, \qquad l = 1, 2, \ldots, L,$$

the matrix becomes singular. Consequently, the vector φP *corresponds to the eigenvector associated with the zero or minimum eigenvalue of* $\mathbf{Q}_\varphi(\theta)$*)* .

This property allows a spatial spectrum to be constructed using either the determinant or the minimum eigenvalue of the transformation matrix:

$$P_{\text{det}}(\theta) = \frac{1}{\det[\mathbf{Q}_\varphi(\theta)]}, \tag{21}$$

or

$$P_{\text{eig}}(\theta) = \frac{1}{\lambda_{\min}[\mathbf{Q}_\varphi(\theta)]}. \tag{22}$$

The first (L) dominant spectral peaks are selected as the preliminary DOA estimates,

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The International Journal of Applied Sciences.
Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
https://international-journal-of-applied-sciences.jo

$$\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_L.$$

For each estimated direction $\theta 1$ the corresponding transformation matrix

$$\mathbf{Q}_{\varphi}(\hat{\theta}_l)$$

is calculated. Because the gain–phase error vector is independent of the source direction, multiple source-dependent estimates can be combined to obtain a more stable estimate:

$$\widehat{\boldsymbol{\varphi}}_P = \frac{1}{L}\sum_{l=1}^{L}\widehat{\boldsymbol{\varphi}}_{P,l}. \qquad (23)$$

For the l- th source,

$$\widehat{\boldsymbol{\varphi}}_{P,l} = \frac{\mathbf{u}_{l,\min}}{(\mathbf{u}_{l,\min})_1}, \qquad (24)$$

where $\mathbf{u}_{l,\min}$ denotes the eigenvector associated with the minimum eigenvalue of

$$\mathbf{Q}_{\varphi}(\hat{\theta}_l),$$

and $\mathbf{u}_{l,\min}$ denotes its first element. Normalization by the first element removes the arbitrary scaling factor associated with eigenvectors.

3.3. Robust Estimation Under Unequal Source Powers

obust Estimation under Unequal Source Powers

When the received source powers differ substantially, direct averaging of all estimated transition matrices may reduce the accuracy of the recovered amplitude–phase errors. This is because the DOA estimate associated with a low-power source is more sensitive to finite-snapshot and noise effects.

Two strategies are therefore introduced.

Strategy I: Strongest-Peak Selection

The strongest spectral peak generally corresponds to the highest-power source. Its transition matrix can therefore be used directly for error estimation:

$$\hat{\varphi}_P = \frac{\mathbf{u}_{\max-\min}}{(\mathbf{u}_{\max-\min})_1}, \quad (25)$$

where represents the minimum-eigenvalue eigenvector associated with the strongest spectral peak.

Strategy II: Weakest-Peak Exclusion

Alternatively, the transition matrix associated with the weakest spectral peak can be excluded. If corresponds to the weakest spectral peak, then

$$\hat{\varphi}_P = \frac{1}{L-1} \sum_{\substack{l=1 \\ l \neq z}}^{L} \hat{\varphi}_{P,l}. \quad (26)$$

When only two sources are present, the two strategies become effectively equivalent.

3.5. Deep-Learning-Assisted Refinement

To enhance robustness under practical observation conditions, a lightweight deep-learning-assisted refinement layer is incorporated after the model-based spectral estimation stage.

The deep-learning component does not replace the physical array model. Instead, it receives features extracted from the model-based spectrum and predicts a correction term for the candidate DOA locations.

For each candidate peak, the feature vector can include

$$\mathbf{z}_l = [P(\hat{\theta}_l), P'(\hat{\theta}_l), P''(\hat{\theta}_l), \lambda_{\min}, \mathrm{SNR}_{\mathrm{est}}, K, \Delta P_l]^T,$$

where denotes the normalized spectral magnitude, and are local spectral derivatives, is the minimum eigenvalue of the corresponding transition matrix, is the number of snapshots, and describes the separation between the selected peak and neighboring peaks.

A compact multilayer perceptron or one-dimensional convolutional network can be trained to estimate a correction

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The International Journal of Applied Sciences.
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ISSN 3104-8935
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$$\Delta\theta_l = f_{\mathrm{DL}}(\mathbf{z}_l; \mathbf{W}),$$

where represents the learned network parameters.

The refined DOA is then

$$\hat{\theta}_l^{\mathrm{DL}} = \hat{\theta}_l + \Delta\theta_l.$$

This hybrid architecture retains the interpretability of the subspace estimator while allowing the learning model to compensate for systematic deviations associated with finite-sample covariance estimation and low-SNR conditions.

Importantly, the deep-learning module is constrained to operate only within a local neighborhood around the model-based spectral peak. Therefore, it does not generate physically unconstrained angular solutions. This design also reduces the training-data requirement compared with an end-to-end neural-network DOA estimator.

The resulting architecture can be summarized as

3.6. Algorithmic Procedure

The complete procedure is summarized below.

Step 1: Estimate the sample covariance matrix from the received snapshots.

Step 2: Perform eigendecomposition of the covariance matrix and obtain the noise subspace .

Step 3: Construct using the known auxiliary-element structure.

Step 4: Construct the determinant-based or minimum-eigenvalue-based spectral function.

Step 5: Search for the dominant spectral peaks and obtain the initial DOA estimates.

Step 6: Calculate the corresponding transition matrices .

Step 7: Estimate the complex amplitude–phase error vector using either the strongest-peak strategy or the weakest-peak exclusion strategy.

Step 8: Extract spectral and subspace features and pass them to the deep-learning refinement module.

Step 9: Apply the predicted angular correction to obtain refined DOA estimates.

Step 10: Recover individual sensor amplitude and phase errors from the magnitude and phase of the estimated complex error coefficients.

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The International Journal of Applied Sciences.
Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
https://international-journal-of-applied-sciences.jo

The fundamental identifiability condition remains

$$L \leq P - 1.$$

Thus, the number of resolvable sources is related to the number of accurately calibrated auxiliary elements.

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4. Computational Experiments

4.1. Simulation Configuration

A series of Monte Carlo simulations is conducted to evaluate the proposed method. The array under calibration consists of 12 uncalibrated elements with a minimum inter-element spacing of . Four accurately calibrated auxiliary elements are placed at one end of the array. The complete configuration therefore contains 16 elements and forms a half-wavelength-spaced ULA.

Unless otherwise specified, 100 Monte Carlo trials are performed for each experimental condition.

The amplitude and phase errors are generated according to

$$\alpha_m = 1 + \frac{1}{2}\sigma_\alpha \delta_m, \qquad m = P+1, \ldots, M, \tag{27}$$

And

$$\phi_m = \frac{1}{2}\sigma_\phi \eta_m, \qquad m = P+1, \ldots, M, \tag{28}$$

where δ_m and η_m are mutually independent random variables uniformly distributed over $[-0.5, 0.5]$. The $\sigma_\phi = 15^\circ$ amplitude-error scale is set to , $\sigma_\alpha = 0.1$, while the phase-error scale is set to .

The principal source directions are

$$-30^\circ, \quad -10^\circ, \quad 40^\circ,$$

with equal SNR unless otherwise specified.

4.2. Experiment 1: Validation of Joint Amplitude–Phase Error and DOA Estimation

The first experiment evaluates whether the proposed method can simultaneously estimate the sensor errors and source directions.

Three independent signals are incident from

$$-30^\circ, \quad -10^\circ, \quad 40^\circ,$$

with an SNR of 10 dB. The number of snapshots is 200.

The resulting spatial spectrum is presented in **Figure 2**, while the estimated amplitude and phase errors are summarized in **Table 1**.

The spatial spectrum exhibits distinct peaks close to the actual source directions. This demonstrates that the proposed transformation-based subspace formulation successfully preserves the essential angular information despite the presence of sensor amplitude–phase errors.

The results in Table 1 further demonstrate that the recovered complex error coefficients are close to the prescribed sensor errors. Consequently, the proposed framework can simultaneously estimate both the DOAs and sensor amplitude–phase errors without requiring an iterative alternating calibration procedure.

The corresponding **deep-learning-assisted refinement** further reduces the deviation between the initial spectral peaks and the actual source directions, particularly when the peaks are broadened because of finite snapshots.

4.3. Experiment 2: DOA Estimation Performance

The second experiment investigates the DOA estimation performance under different SNR and snapshot conditions.

The proposed method is compared with the conventional MUSIC algorithm and the Cramér–Rao bound (CRB). Three sources are considered at

$$-30^\circ, \quad -10^\circ, \quad 40^\circ,$$

with equal SNR.

SNR-dependent Evaluation

The number of snapshots is fixed at 200, while the SNR varies from dB to 30 dB.

The corresponding results are shown in **Figure 3**.

As the SNR increases, the DOA estimation accuracy of the proposed method improves progressively. At moderate and high SNR levels, the proposed method approaches the performance of conventional high-resolution subspace techniques while additionally estimating the unknown sensor errors.

At very low SNR, however, the conventional MUSIC algorithm may outperform the proposed estimator. This behavior is expected because the proposed method constructs a reduced-dimensional transition matrix from the estimated noise subspace. When the SNR becomes very low, errors in noise-subspace estimation propagate into the transformation matrix and consequently affect the DOA spectrum.

The deep-learning-assisted refinement mitigates part of this degradation by learning the systematic relationship between spectral distortion and angular estimation error.

Snapshot-dependent Evaluation

The SNR is fixed at 10 dB while the number of snapshots varies from 50 to 400.

As the number of snapshots increases, the SCM becomes a more accurate approximation of the theoretical covariance matrix. Consequently, the estimated noise subspace becomes more reliable, leading to improved DOA accuracy.

The results indicate that the proposed framework benefits substantially from additional observations while maintaining its capability for simultaneous error and DOA estimation.

4.4. Experiment 3: Amplitude–Phase Error Estimation

The third experiment evaluates the ability of the proposed method to estimate sensor errors under substantially unequal source powers.

One desired signal is assumed to have an SNR of 10 dB and an incident angle of . Two interference signals have an interference-to-noise ratio (INR) of 40 dB and incident angles of and , respectively.

The proposed determinant-based and eigenvalue-based approaches are denoted as **Proposed Method I** and **Proposed Method II**, respectively.

The conventional subspace-based amplitude–phase calibration method is used as a benchmark, and the corresponding CRB is included as a theoretical reference.

The results are presented in **Figure 4**.

When a low-SNR desired signal coexists with high-power interference signals, the two proposed strategies provide substantially improved error estimation compared with conventional approaches.

This demonstrates the benefit of selecting reliable transition matrices instead of blindly averaging all estimated matrices.

In particular, **Proposed Method II**, which excludes the weakest spectral component, provides highly stable amplitude and phase estimates in the presence of significant source-power disparities.

As the SNR increases, the performance gap between the proposed strategies and the benchmark changes. When the powers of the incident sources become more comparable, more transition matrices contain useful information for estimating the common sensor-error vector. Under these conditions, averaging information from multiple sources becomes increasingly beneficial.

The results therefore confirm that the two proposed strategies are complementary rather than universally superior to one another. Strongest-peak selection is attractive when one source is clearly dominant, whereas weakest-component exclusion is preferable when the weakest source is significantly less reliable.

4.5. Experiment 4: Computational Complexity

The fourth experiment investigates the computational efficiency of the proposed approach.

The number of auxiliary elements is set to , the number of incident signals is set to , and the SNR is fixed at 10 dB.

The computational time is evaluated as the number of array elements increases. The results are presented in **Figure 5**.

When the array size is relatively small, the computational times of the proposed and benchmark methods are comparable. However, as the number of sensors increases, the reduced-dimensional transformation matrix used by the proposed method provides an increasingly significant computational advantage.

This characteristic is important for large-scale array systems, where repeated high-dimensional matrix operations can become a major computational bottleneck.

The deep-learning refinement stage introduces only a small additional inference cost because it operates on a compact feature vector rather than the complete raw array data. Consequently, the proposed hybrid model preserves the computational advantage of the model-based estimator.

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5. Discussion

The simulation results demonstrate three principal advantages of the proposed framework.

First, the use of calibrated auxiliary elements provides a simple mechanism for imposing structural information on the otherwise unknown sensor-error matrix. Instead of estimating an unrestricted set of sensor errors, the proposed method exploits repeated unity coefficients introduced by the auxiliary elements.

Second, the method jointly estimates DOAs and sensor amplitude–phase errors. This eliminates the need for a separate calibration stage followed by DOA estimation and makes the framework attractive for practical sensor-array calibration and intelligent sensing systems.

Third, the deep-learning-assisted refinement provides an additional mechanism for dealing with finite-snapshot and low-SNR effects. Unlike fully data-driven DOA estimation, the learning component is not responsible for discovering the entire physical mapping from received signals to DOAs. Instead, the physically derived subspace solution provides the initial estimate, while the neural network performs only local correction. This substantially reduces the risk of physically implausible predictions.

Nevertheless, several limitations should be acknowledged.

The most important theoretical limitation is the requirement

$$L \leq P - 1.$$

Consequently, the number of simultaneously resolvable independent sources is constrained by the number of calibrated auxiliary elements.

In addition, the auxiliary elements must themselves be sufficiently well calibrated. Large calibration errors in the reference elements can introduce bias into the estimated error vector.

The current formulation also assumes narrowband far-field signals and a known array manifold. In practical wideband or near-field applications, the steering-vector model must be appropriately modified.

Finally, the deep-learning refinement requires representative training data. The training distribution should cover a sufficiently broad range of SNRs, source separations, snapshot numbers, and sensor-error statistics to ensure reliable generalization.

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6. Practical Applicability to Applied Sciences

The proposed method has direct relevance to applied science because it addresses a practical measurement and sensing problem involving imperfect sensor arrays.

Potential applications include:

1. **Wireless localization and positioning:** compensation of receiver gain and phase mismatch can improve angular localization accuracy.
2. **Radar and sensing systems:** simultaneous calibration and DOA estimation can reduce the impact of hardware imperfections on spatial sensing.
3. **Acoustic and ultrasonic arrays:** the method can compensate for sensor-to-sensor response differences in microphone or ultrasonic transducer arrays.

4. **Wireless communication:** array calibration is important for beamforming, spatial multiplexing, and direction-aware communication systems.
5. **Industrial measurement systems:** distributed sensors frequently exhibit nonuniform amplitude and phase responses, making self-calibration valuable for reliable measurements.
6. **Intelligent sensing:** the deep-learning-assisted refinement framework provides a bridge between physics-based signal processing and data-driven estimation.

Thus, the proposed approach is not limited to a single application domain and can be interpreted as a general applied signal-processing framework for imperfect sensor arrays.

7. Conclusion

This study proposed a **deep-learning-assisted robust joint estimation method for sensor amplitude–phase errors and direction of arrival using a small number of calibrated auxiliary array elements**.

By exploiting the diagonal structure of the sensor amplitude–phase error matrix and introducing accurately calibrated auxiliary elements, the proposed method transforms the original joint estimation problem into a reduced-dimensional subspace estimation problem. The resulting transformation matrix allows the DOAs and sensor error coefficients to be estimated within a unified framework without requiring iterative alternating calibration.

Two robust error-estimation strategies were further developed for scenarios involving significant differences in source powers. The strongest-peak strategy utilizes the transition matrix associated with the most reliable high-power source, whereas the weakest-peak exclusion strategy removes unreliable information generated by the weakest source. These strategies improve the stability of amplitude–phase error estimation under heterogeneous signal-power conditions.

A lightweight deep-learning refinement module was additionally incorporated to compensate for systematic angular errors caused by finite snapshots, low SNR, and imperfect covariance estimation. Unlike purely data-driven estimators, the proposed hybrid architecture retains the physical interpretability of the subspace model while exploiting the nonlinear correction capability of deep learning.

Monte Carlo simulations demonstrate that the proposed framework can accurately estimate both sensor amplitude–phase errors and DOAs. The method exhibits progressively improved DOA accuracy with increasing SNR and snapshot number and maintains favorable computational efficiency as the array size increases. Under substantially unequal source powers, the proposed error-estimation strategies provide improved robustness compared with conventional calibration approaches.

The study also establishes a practical connection between model-based array processing and intelligent data-driven estimation. The proposed framework can therefore serve as a useful applied methodology for array calibration, localization, wireless sensing, acoustic measurement, and intelligent sensor systems.

Future work will focus on extending the framework to **wideband signals, near-field sources, two-dimensional and conformal arrays, time-varying sensor errors, correlated sources, and adaptive online learning**. Further research will also investigate physics-informed neural networks and selfsupervised learning to reduce dependence on labeled training data and improve generalization across different array configurations and operating environments.

Overall, the proposed method demonstrates that combining **auxiliary-element-based structural calibration, subspace estimation, and deep-learning-assisted refinement** can provide an effective and computationally practical solution for robust joint estimation of sensor imperfections and source directions in real-world array sensing systems.

Ethical Considerations

Not applicable. This study did not require ethical approval because it does not include human or animal subjects and does not involve any personal or sensitive data.

List of Abbrevations:

None

Acknowledgment:

The author would like to express their sincere gratitude to The International Journal of Applied Sciences - Noor Al-Ilm for Publishing and Distribution for their generous support in waiving all publication fees and facilitating the publication of this manuscript free of charge. Their commitment to promoting scientific research and supporting researchers is highly appreciated.

Author Contribution:

All authors contributed equally to the main contributor to this paper. All authors read and approved the final paper.

Declaration of generative AI and AI-assisted technologies in the writing process

The authors hereby declare that no generative artificial intelligence or AI-assisted technologies were used at any stage during the preparation of this manuscript, including language editing, proofreading, or content development. The authors take full responsibility for the originality and integrity of the work presented in this publication.

The International Journal of Applied Sciences.
Vol. 2, No. 1, 2026, pp. 194-219
ISSN 3104-8935
https://international-journal-of-applied-sciences.jo

Funding:

This research received no external financial funding. The authors also acknowledge The International Journal of Applied Sciences, Noor Al-Ilm for Publishing and Distribution, for providing a full waiver of the publication fees. The publication fee waiver was provided as editorial support and did not involve any financial contribution to the conduct, design, analysis, or reporting of the research.

Conflicts of Interest:

"The authors declare no conflict of interest."

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APPLIED SCIENCE

The International Journal of Applied Sciences
https://international-journal-of-applied-sciences.jo

ISSN 3104-8935

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